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ON G. I. BARENBLATT'S RESPONSE ABOUT THE PAPER "ILLEGITIMATE TENDENCIES IN THE USE OF THE CONCEPT OF SELF-SIMILAR PHENOMENA"*

V. V. MARKOV

In connection with the response of G. I. Barenblatt published in PMM, No.2, which contains direct errors, attention must be turned to the following remarks.

1°. There exist and are considered formulations of problems by different authors which are independent of any other problems whose solutions are self-similar.

2°. Investigation of the formulation and solution of nonself-similar problems for which this self-similar solution can be approximate in some theoretical or practical sense and in appropriate ranges are questions for which the answers are generally distinct, they are related to the properties of the formulation of nonself-similar problems and with the behavior of their solutions. Such answers can evidently be just a characteristic of the appropriate fixed nonself-similar problems or of different sequences of nonself-similar solutions determined separately and independently of this self-similar solution which has been established exactly.

3°. In the supplement to the remarks in my paper and section 1° and 2° we prove that besides the absence of any need from the scientific viewpoint to separate self-similar solutions into solutions of the first and second kind, as given in the book of G. I. Barenblatt and repeated in his response, it is unacceptable in substance too.

In fact, according to the π-theorem, let the relationship (**)

$$\pi = \Phi(\pi_1, \pi_2, \ldots, \pi_m) \tag{1}$$

hold for the solution of nonself-similar problems.

The limit equality is examined: for $\pi_1 \to \overset{0}{\infty}$.

The definition of Barenblatt is: If $\Phi(\overset{0}{\infty}, \pi_2, \ldots, \pi_m) \neq \overset{0}{\infty}$, , then a self-similar solution of the first kind is obtained in the limit from (1)

$$\pi = \Phi_1(\pi_2, \ldots, \pi_m) \tag{2}$$

If as $\pi_1 \to \overset{0}{\infty}$

$$\pi = \Phi(\pi_1, \pi_2, \ldots, \pi_m) \to \pi_1^\alpha \Phi_1(\pi_2, \ldots, \pi_m)$$

where $\Phi_1(\pi_2, \ldots, \pi_m) \neq \overset{0}{\infty}$, then (1) yields the limit relationship

$$\pi = \pi_1^\alpha \Phi_1(\pi_2, \ldots, \pi_m) \tag{3}$$

which Barenblatt rewrites in the form

$$\frac{\pi}{\pi_1^\alpha} = \pi_* = \Phi_1(\pi_2, \ldots, \pi_m) \tag{4}$$

The relationship (3) is called a self-similar solution of the second kind.

The relationships (3) and (4) are evidently indistinguishable, but (4) in the variables $\pi_*, \pi_2, \ldots, \pi_m$ has the identical form and meaning as (2) in the variables $\pi, \pi_2, \ldots, \pi_m$. Hence, (4) falls into the definition of a self-similar solution of the first kind, and moreover, is another way of writing the same solution defined by (3) and called a self-similar solution of the second kind.

Hence, it follows that the separation given by Barenblatt for self-similar solutions is meaningless. The presence of the unknown exponent α in the definition of π_* does not alter the crux of the matter.

It should be added and emphasized that the question of setting up exponents of the type α which are unknown in advance, for the self-similar solutions by mathematical considerations during the solution of the problem or from tests has been discussed in detail for turbulent motions in the first 1944 edition of the book of L. I. Sedov and for the theory of waves in all the subsequent editions, and before this the exponent α was determined during the solution of the problem by Guderley and by other authors in deriving original results.

Let us emphasize that the definitions described are not constructive since the functions (1) and (3) are understandably already known from the solution of nonself-similar problems, but the construction and utilization of self-similar solutions are not required in the presence of such solutions.

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**) The notation, formulas, and numbering are taken from the published response of Barenblatt.

4°. In contrast to the declaration of Barenblatt, trivial solutions of the problems of filtration and explosion with energy evolution by a shock exist. The fact is that these solutions have meaning for $t>0$ but not for $t\geqslant 0$ as is indicated in the response. In the general case, the quantity Q in (3.4) for the solution of the problem of an explosion with radiation depends on time for $t\neq 0$. Because of the properties of the problem when passing to the limit, $Q\neq 0$ for $t=0$ and $Q=0$ for $t>0$. In the solution given in the book of Barenblatt, $Q=\infty$ for $t=0$.

5°. A nontrivial solution with finite Q and conditions physically well-founded on the discontinuity is given in the Appendix.

6°. In connection with the paper by Ia. G. Sapunkov, the response of Barenblatt is unsatisfactory. His assertion about the intersection of his results and the results of Sapunkov for $\gamma_1=2\gamma+1$, and about the degeneration of the integral curve mentioned in the paper are erroneous. The fact is that Barenblatt considers the "phase portrait is a picture of the integral curves of equation (4.18)" dependent on γ_1 (pp. 72-74 in Barenblatt's book) while neither (4.18) nor the parabola (4.24), which are the "geometric locus of points of the front" are independent on γ_1. The field of integral curves of (4.18) is determined just by the quantities α and γ. Following the usual approach of the "first kind", a whole class of self-similar problems with the variable r/t^{α} and with different boundary conditions can be considered for (4.18), and solutions of the problem studied by Barenblatt, or the Ia. G. Sapunkov problem of a detonation wave subjected to the Chapman–Jouguet condition can be obtained as a particular case. Hence, by giving the value of α and finding the point of intersection of the integral curve issuing from the image of the center of symmetry and the parabola (4.24) in the zV plane, the dependence $\alpha(\gamma_1)$ can be constructed by using the relationships (4.24) which yield a dependence of the coordinates z and V of the image of the front on α and γ_1. Using his incorrect approach, Barenblatt omits a whole continuum of solutions from the Chapman–Jouguet wave $(\gamma_1=2\gamma+1)$ corresponding to the values of $\alpha:(3\gamma+3)/(5\gamma+3)<\alpha<1$. Fig. 4.3 of his book lacks the corresponding integral curve starting from the point of intersection of the parabola (4.24), the image of the front, and the curve corresponding to weak discontinuities. It is easy to see that the section of the integral curve that degenerates into a point for $\gamma_1=2\gamma+1$ according to the assertion of Barenblatt, is completely finite. It cannot be otherwise since the field of integral curves is independent of γ_1. Consequently, Fig. 4.2, illustrating the dependence $\alpha(\gamma_1)$ is also false. There is no vertical section from $\alpha=(3\gamma+3)/(5\gamma+3)$ to $\alpha=1$, at the point $\gamma_1=2\gamma+1$ which corresponds to solutions with a Chapman–Jouguet wave.

Evidently, the intersection of the results of Barenblatt and Sapunkov cannot be spoken of. Namely, the nonuniqueness of $\alpha(\gamma_1)$ is indicated in the paper for $\gamma_1=2\gamma+1$, which, as is easy to see from Fig.4.2 in the book, Barenblatt left out although he tries to assert in the response that there is such a nonuniqueness there.

7°. Questions of priority associated with the papers of Beckert and Guderley are also resolved by Barenblatt contrary to the truth. There is apparently no sense repeating what these authors did in the area of self-similar solutions. There remains just the recommendation of careful examination of the material cited by Barenblatt to understand the erroneousness of this assertion. The assertion that the Guderley solution is based on dimensional analysis is not true.

8°. As regards the question of turbulence, Barenblatt here distorts the crux of the matter and speaks about local isotropic turbulence while in the paper we speak and cite literature only on complete isotropic turbulence.

Appendix. Self-similar solution of the heat conduction equation with a discontinuous coefficient. Let the heat conduction process be described by an equation with the piecewise-constant coefficient $\varkappa$ (See, Barenblatt, Similarity, Self-similarity, Intermediate Asymptotics. Gidrometeoizdat, Leningrad, 1978).

$$\frac{\partial T}{\partial t}=\varkappa_1\frac{\partial^2 T}{\partial x^2}\left(\frac{\partial T}{\partial t}\leqslant 0\right),\quad \frac{\partial T}{\partial t}=\varkappa\frac{\partial^2 T}{\partial x^2}\left(\frac{\partial T}{\partial t}\geqslant 0\right)$$

Let the problem of heat propagation because of delivery of a finite quantity of heat to $x=0$ at $t=0$ be considered for this equation under the assumptions that the change in sign of $\partial T/\partial t$ occurs once, and that $T=T(x,0)=0$ at the initial instant, and there is no heat influx for $x=\pm\infty$ and $t\geqslant 0$.

This problem is evidently self-similar (with the self-similar variable $\xi=x/\sqrt{\varkappa t}$) and the quantity of heat is constant between any two movable planes on which $\xi=\text{const}$. Hence, by giving the value $\xi=\xi_0$ on the discontinuity $\varkappa$ and constant quantities of heat $Q_1>0$ for $0\leqslant\xi\leqslant\xi_0$ and $Q_2>0$ for $\xi_0\leqslant\xi<\infty$, the solution which has the form

$$T=\frac{1}{\sqrt{\varkappa t}}Q_1\left\{2C_{v1}\int_0^{\xi_0}\exp\left(-\frac{\xi^2}{4\varepsilon}\right)d\xi\right\}^{-1}\exp\left(-\frac{\xi^2}{4\varepsilon}\right)\quad(0\leqslant\xi\leqslant\xi_0)$$

$$T = \frac{1}{\sqrt{\varkappa t}} Q_2 \left\{ 2C_v \int_{\xi_0}^{\infty} \exp\left(-\frac{\xi^2}{4}\right) d\xi \right\}^{-1} \exp\left(-\frac{\xi^2}{4}\right) \quad (\xi_0 \leqslant \xi < \infty) \quad \varepsilon = \varkappa_1/\varkappa \qquad (*)$$

is determined completely, as is easy to verify.

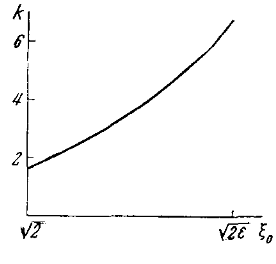

Fig.1

In conformity with the requirement of a single change in the sign of $\partial T / \partial t$, the quantity ξ_0 should satisfy the condition $\sqrt{2} \leqslant \xi_0 \leqslant \sqrt{2\varepsilon}$, and the solution exists for $\varepsilon \geqslant 1$. It is seen from the solution (*) that the total energy flux at the point of discontinuity $\varkappa$ is zero, and the functions T and $\partial T / \partial x$ undergo discontinuities in the general case which are determined by the quantities $\xi_0, Q_1, Q_2, C_{v1}, C_v, \varkappa$ and $\varkappa_1$ from the formulas (*).

Since the solution is not determined uniquely, it can be required in addition that the temperature at the point of the discontinuity $\varkappa$ be continuous. This condition yields a relationship between $k = (Q_1 C_v)/(Q_2 C_{v1})$ and ξ_0

$$k = \sqrt{\varepsilon}\,\Phi\left(\frac{\xi_0}{2\sqrt{\varepsilon}}\right) \Phi'\left(\frac{\xi_0}{2}\right) \left\{ \Phi'\left(\frac{\xi_0}{2\sqrt{\varepsilon}}\right) \left[\sqrt{\pi} - \Phi\left(\frac{\xi_0}{2}\right)\right] \right\}^{-1}, \quad \left(\Phi(z) = 2\int_0^z \exp(-z^2)\, dz\right)$$

A graph of this dependence is represented in the Fig.1 for $\varepsilon = 4$. Let us note that the ratio between its gradients to the right and left of the discontinuity is constant, equal to $1/\varepsilon$, under the condition of continuity of the temperature.

Translated by M.D.F.